CC. Talk: Paris, 6/15/2018

Progress on the Cosmological Constant

Λ

With P. Graham and S. Rajendran

What is the cosmological constant problem?

See Nemanja's talk

What is the cosmological constant problem?

Λ

est.

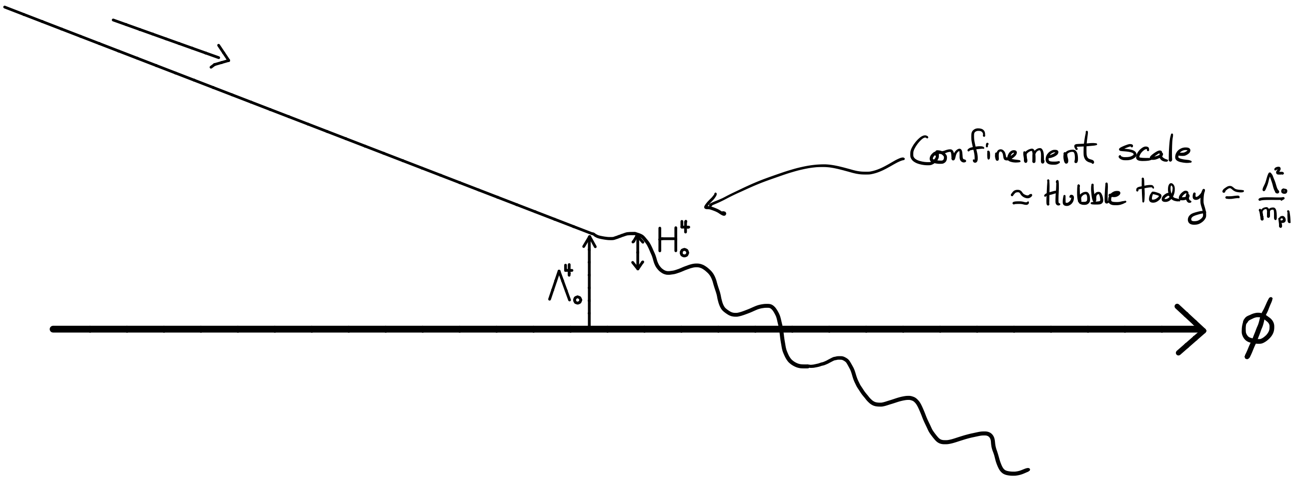
Λ

obs.

To solve dynamically:

- $\frac{\Lambda_{cc}^2}{M_{pl}} \sim H_0 \sim 10^{-33} eV$ is a very small energy
  - suggests it will take a long time (at least H_0^{-1}) to dynamically adjust
- Only gravity senses the C.C. $\to$ must be involved in stopping the evolution.
- Gravity couples to all energy universally $\to$ evolution stops when all energy density is small.

The Abbott Model $\quad \mathcal{L} = k^3\phi + \frac{\phi}{f} G\tilde{G} + \frac{1}{g^2} GG$



ϕ slow rolls $\longrightarrow$ Inflates & stops when curvature is H_0

The Universe is empty

Tiny slope so H_0^4/f can stop it $\longrightarrow$ quantum fluctuations dominate

Eternal Inflation

Why is Abbott so great?

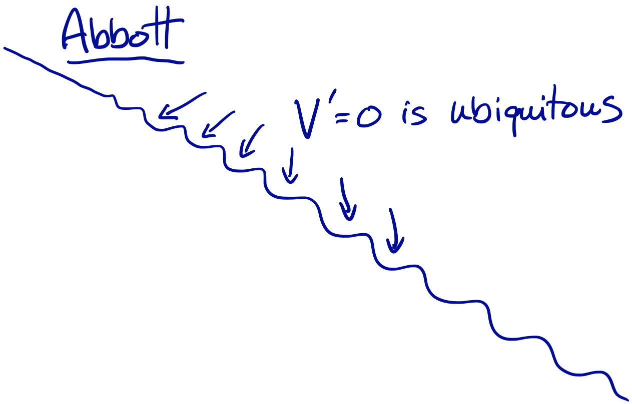
Weinberg's no-go:

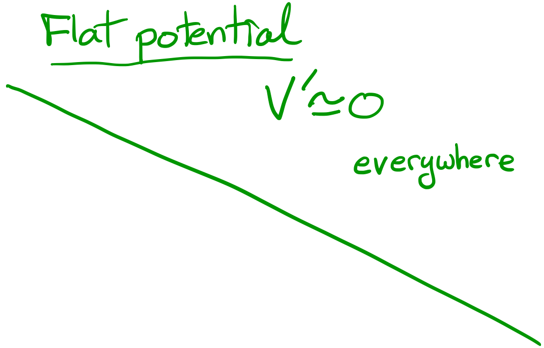
$V' = 0$ & $V \simeq 0$ at the same point is a tuning.

Why is Abbott so great?

Weinberg's no-go:

$$V' = 0 \quad \& \quad V \simeq 0$$





Fix Abbott

- Roll to a tiny C.C. with a steeper slope.
- Reheat empty Universe by violating energy conservation.
  - simplest way : gravity (Weinberg - Witten)

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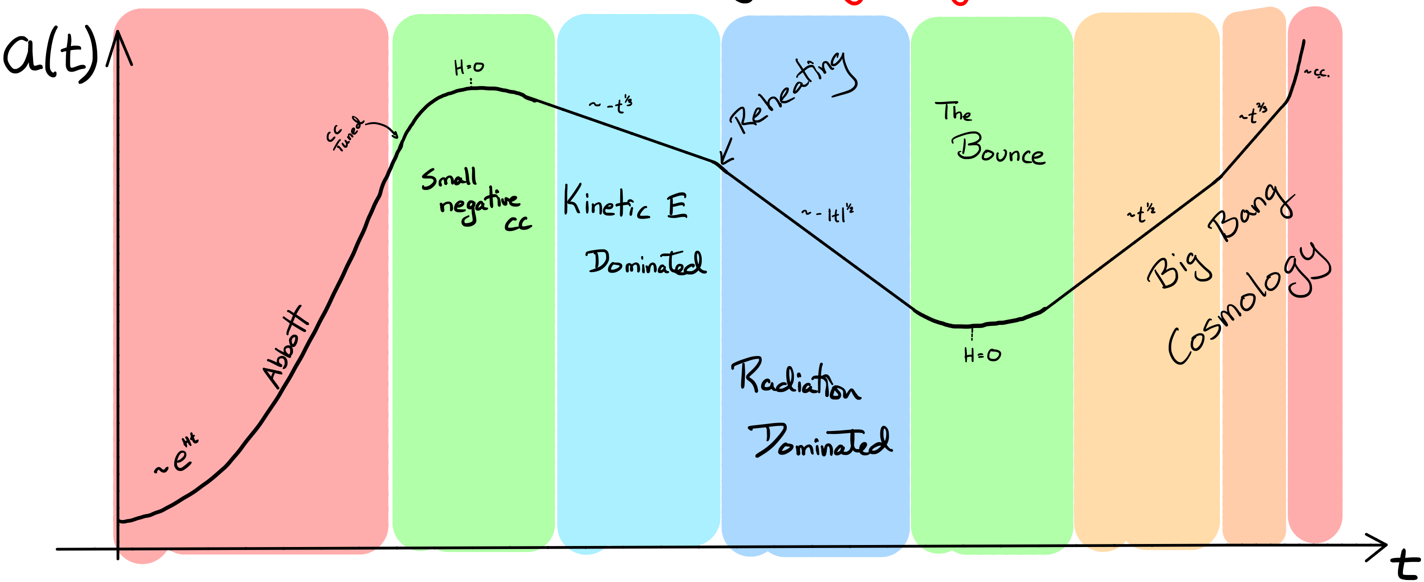
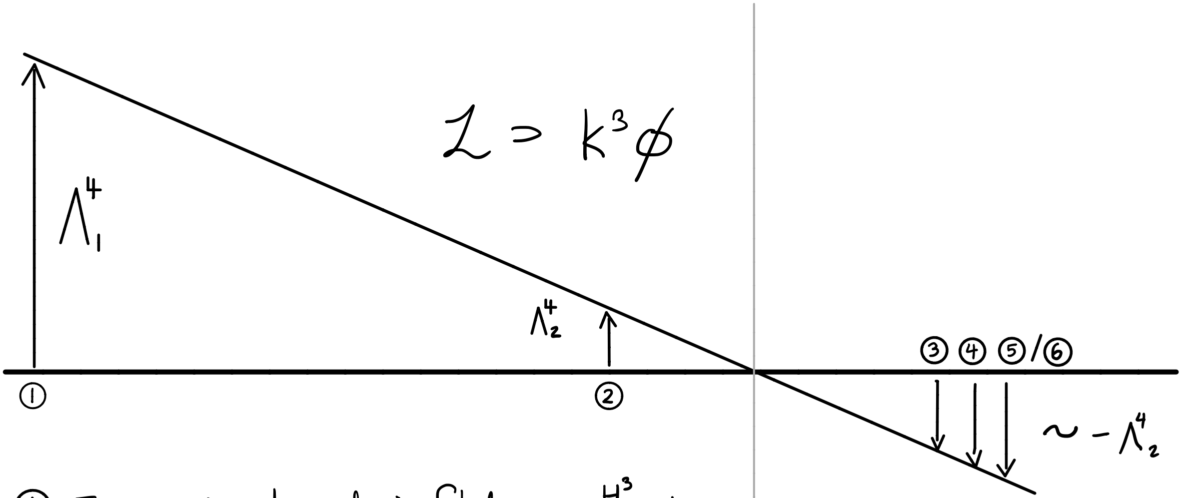


The Roll



① To avoid eternal inflation, $\frac{H^3}{V'} < 1$
(assuming slow roll) $\rightarrow$ $\boxed{k > \Lambda_1^2 / m_{pl}}$

② Beginning fast roll $\longrightarrow$ $\boxed{k^3 m_{pl} \simeq \Lambda_2^4}$

① + ② $\longrightarrow$ $\boxed{\Lambda_1^3 < \Lambda_2^2 m_{pl}}$

for $\Lambda_2 \sim \text{meV}$, $\Lambda_1 < 10\,\text{MeV}$

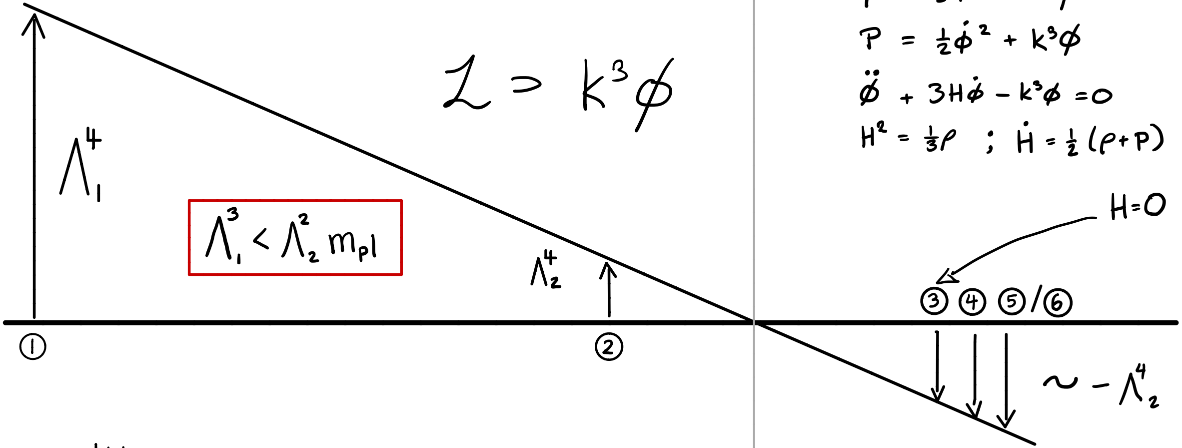
The Roll

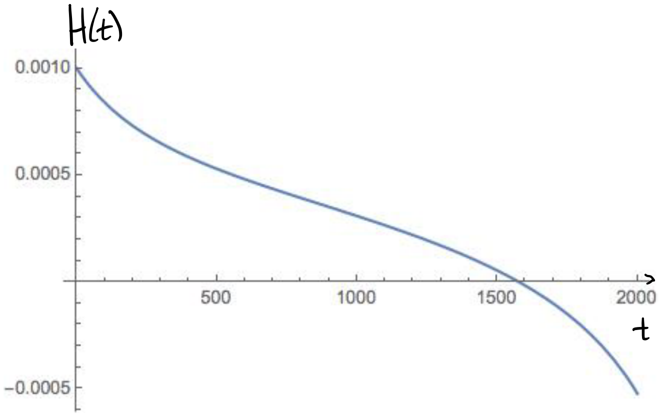
③ Roll until $H=0$

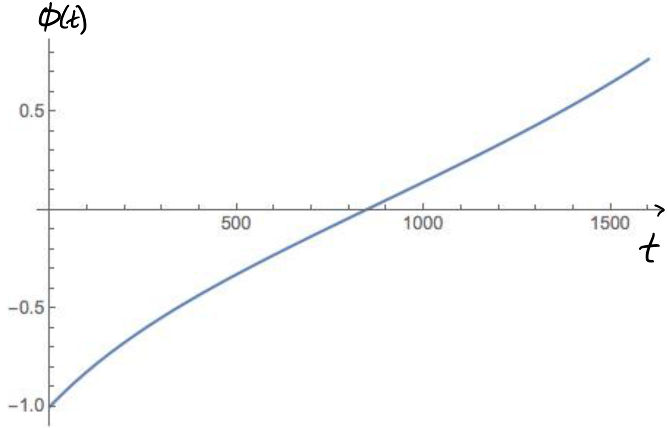
$$\rho = \tfrac{1}{2}\dot{\phi}^2 - k^3\phi$$
$$P = \tfrac{1}{2}\dot{\phi}^2 + k^3\phi$$
$$\ddot{\phi} + 3H\dot{\phi} - k^3\phi = 0$$
$$H^2 = \tfrac{1}{3}\rho \;\; ; \;\; \dot{H} = \tfrac{1}{2}(\rho + P)$$

Initial Conditions

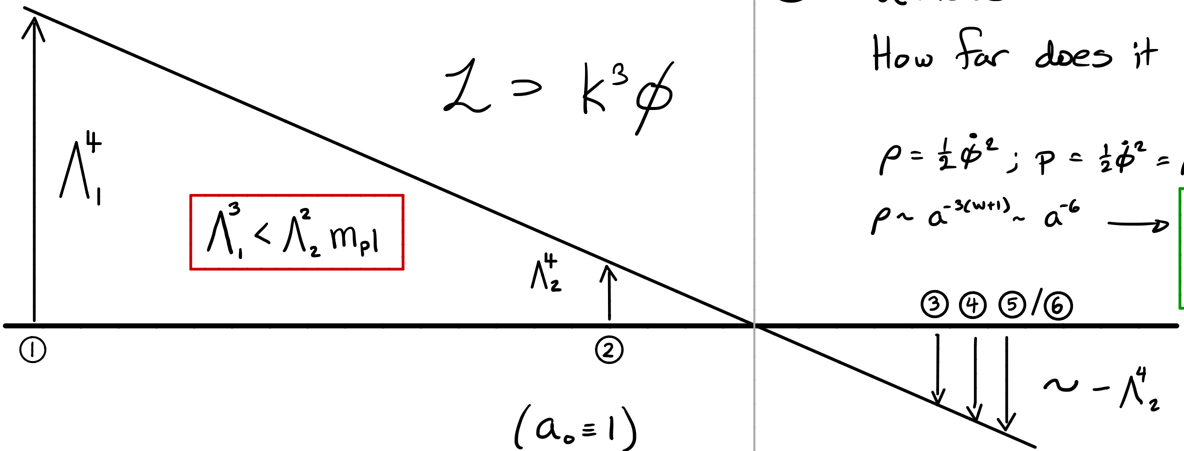
$$\phi_2 = -1$$
$$\dot{\phi}_2 \sim \Lambda_2^2$$
$$H_2^2 = \tfrac{1}{3}\rho_2$$







The Roll



$$\mathcal{L} \supset k^3 \phi$$

$$\boxed{\Lambda_1^3 < \Lambda_2^2 m_{pl}}$$

$$\dot\phi(t) = \dot\phi(a(t)) = \dot\phi_0 \frac{1}{a(t)^3}$$

$$H = \frac{\dot a}{a} = -\frac{1}{\sqrt{3} m_{pl}} \dot\phi_0 \frac{1}{a^3}$$

$$\rightarrow a^2 da = -\frac{\dot\phi_0}{\sqrt{3} m_{pl}} dt$$

④ Universe contracts, $\dot\phi$ dominates

How far does it roll?

$$\rho = \tfrac{1}{2}\dot\phi^2 \,;\; P = \tfrac{1}{2}\dot\phi^2 = \rho \quad (w=1)$$

$$\rho \sim a^{-3(w+1)} \sim a^{-6} \longrightarrow \boxed{\dot\phi \sim \frac{1}{a^3}}$$

$$\Delta\phi = \int \dot\phi \, dt$$

$$= \int \dot\phi_0 \frac{1}{a^3} \left(\frac{-\sqrt{3} m_{pl}}{\dot\phi_0} \right) a^2 da$$

$$\boxed{\Delta\phi = \sqrt{3} m_{pl} \log(1/a)}$$

Only logarithmic

The Universe contracts as ϕ rolls

→ Reheat: add coupling $\boxed{\mathcal{L} \supset \frac{\phi}{f} F' \tilde{F}'}$ to U(1) gauge boson, mass $m_{A'}$

E.O.M.: $\ddot{\phi} + 3H\dot{\phi} - \frac{\nabla^2}{a^2}\phi + V'(\phi) = \frac{1}{f} \vec{E}\cdot\vec{B}$ $\quad (\vec{E} = \dot{\vec{A}},\ \vec{B} = \frac{1}{a}\vec{\nabla}\times\vec{A})$

$$\left(\partial_t^2 + H\partial_t - \frac{\nabla^2}{a^2} - m_{A'}^2 - \frac{\dot{\phi}}{f}\frac{\vec{\nabla}}{a}\times\right)\vec{A}' = 0$$

→ Spatially Fourier-Transform $\vec{A}'$ with polarizations $\vec{\epsilon}_\pm \cdot \vec{k} = 0$

$\Rightarrow A_\pm(k)$ $\quad \vec{\epsilon}_\pm \times \vec{k} = \mp i |\vec{k}| \vec{\epsilon}_\pm$

For small H vs. m_A, k

$$\ddot{A}_\pm + \left(m_{A'}^2 + k^2 \mp \frac{\dot{\phi}}{f} k\right) A_\pm = 0$$

When $\frac{\dot{\phi}}{f} \gtrsim m_{A'}$, A_+ has tachyonic modes

Anber, Sorbo

For small H vs. m_A, k

$$\ddot{A}_\pm + \left(m_{A'}^2 + k^2 \mp \frac{\dot\phi}{f} k\right) A_\pm = 0$$

- When $\frac{\dot\phi}{f} \gtrsim m_{A'}$, A_+ has tachyonic modes

- numerically (for initial $A_+ \sim k$), $\dot\phi$ loses $\mathcal{O}(1)$ fraction in $\Delta t \sim \mathcal{O}(10-100)\, f/\dot\phi$

  Since initially, $H \sim \dot\phi / m_{pl}$, this is fast if

$$f < \frac{m_{pl}}{\mathcal{O}(10-100)}$$

- Hubble can keep increasing as long as $\mathcal{O}(10-100)\,\frac{f}{\dot\phi} \sim \frac{\mathcal{O}(10-100)}{m_{A'}} \lesssim \frac{1}{H_{rh}}$

reheat

- Hubble can keep increasing as long as $\mathcal{O}(10\text{-}100)\,\frac{f}{\dot{\phi}} \sim \frac{\mathcal{O}(10\text{-}100)}{m_{A'}} \lesssim \frac{1}{H_{rh}}$

  reheat

- Final reheat requirement

  $\longrightarrow$ A' decays to SM in time longer than $\Delta t_{decay} \sim \frac{m_{pl}}{m_{A'} f}$

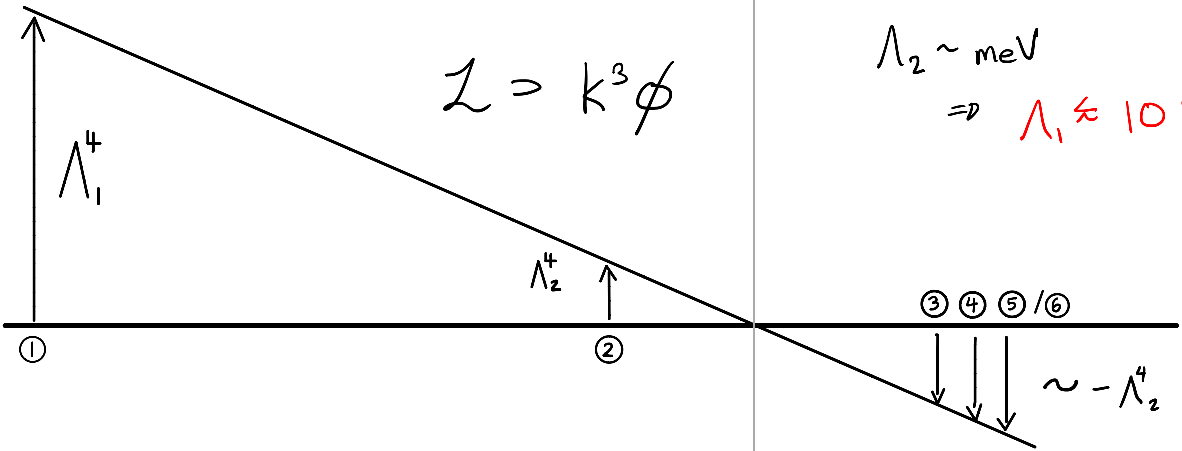
The Roll

$$\boxed{\Lambda_1^3 < \Lambda_2^2 m_{pl}}$$

$\Lambda_2 \sim \text{meV}$

$\Rightarrow \Lambda_1 \lesssim 10\ \text{MeV}$ cutoff

$$\mathcal{L} \supset k^3 \phi$$



5 Universe has contracted and reheated & now needs a bounce.

6 After a bounce, and expansion, $\dot{\phi} = \dot{\phi}_{rh}\, 1/a^3$

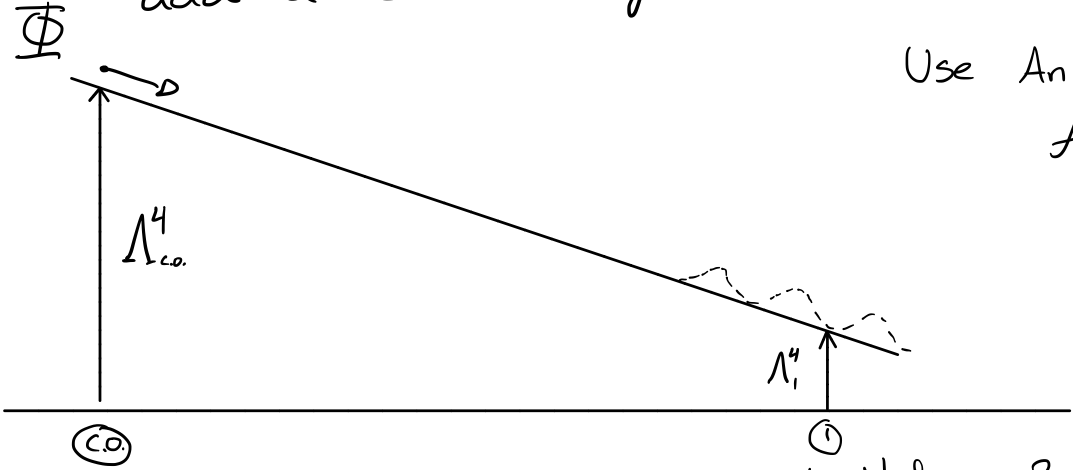
$$\Delta\phi \simeq \int \dot{\phi}_{rh} \frac{dt}{a^3} \simeq \int \dot{\phi}_{rh} \frac{da}{a^2} \frac{1}{H_{rh}}$$

taking $a_{rh} = 1$

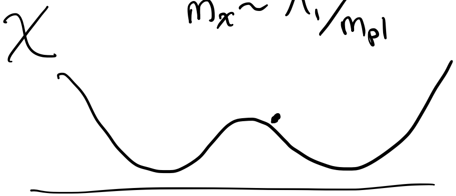
$$= \left(1 - \frac{1}{a_0}\right) \sqrt{3}\, m_{pl}$$

For a higher cutoff,

add a second stage:



Use Anber-Sorbo friction



Can build a 2-field model where χ is stuck until Φ gets to ①, & χ rolls and produces barriers (more to come).

Bouncing in flat FRW:

$$H^2 = \frac{8\pi}{3} G_N \rho \quad ; \quad \dot{H} = -4\pi G_N (\rho + p)$$

To get $H = 0$ & $\dot{H} > 0$

$\rho = 0, \quad p < 0$

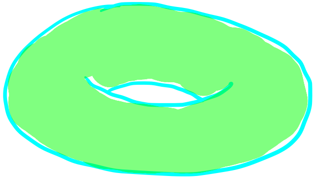
near minimum $(\rho + p) < 0 \Rightarrow w < -1$

Violates the NEC ($n^\mu n^\nu T_{\mu\nu} \geq 0$ for any null n^μ)

NEC can be violated in a Compact space:

Casimir in compact dimensions

$\mathrm{Mink}^4 \times T^2$

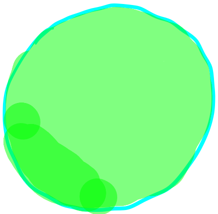


massless fields

$$T_{\mu\nu} = \rho \begin{pmatrix} -1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & -2R^2 & \\ & & & & & -2R^2 \end{pmatrix}$$

$$g^{\mu\nu} = (-1, 1, 1, 1, \tfrac{1}{R^2}, \tfrac{1}{R^2})$$

Positive Curvature is effective NECv



3-Sphere

$$H^2 = \frac{8\pi}{3} G_N \left(\rho - \frac{k}{a^2}\right)$$

$$\dot{H} = -4\pi G_N \left(\rho + p - \frac{2}{3}\frac{k}{a^2}\right)$$

Can produce positive $\dot{H}$

Bouncing $\equiv$ Converging to Diverging Geodesics

Raychaudhuri's Eq. for null geodesics:

$$\frac{d\hat{\theta}}{d\lambda} - \frac{1}{2}\hat{\theta}^2 - 2\hat{\sigma}^2 + 2\hat{\omega}^2 - 8\pi G_N T_{\mu\nu} U^\mu U^\nu$$

U^μ a congruence ; $\hat{\theta} \equiv \hat{g}_{\mu\nu} \nabla^\mu U^\nu$

To get $\dfrac{d\hat{\theta}}{d\lambda} > 0$,

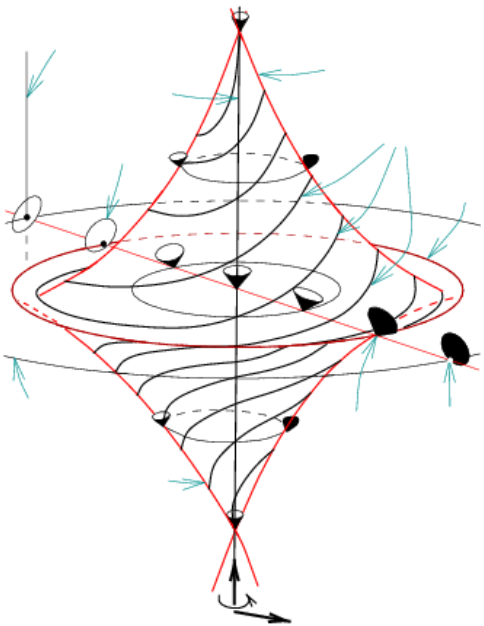
$$T_{\mu\nu} U^\mu U^\nu < 0 \quad (\text{NECv})$$

or

$$\hat{\omega}^2 \neq 0 \Rightarrow \text{VORTICITY}$$

Example of Vorticity : Gödel Metric

$$ds^2 = \frac{2}{\omega^2}\left(-dt^2 + dr^2 + dy^2 - (\sinh^4 r - \sinh^2 r)\,d\phi^2 - 2\sqrt{2}\,\sinh^2 r\; d\phi\, dt\right)$$



Cosmological Const.

+ Spinning Dust

Stationary

→ Closed Timelike Curves

Due to tipping of light cones.

→ Put in compact Space

A Bounce Metric

$$ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2 + L^2 \left(d\theta^2 + d\phi_1^2 + d\phi_2^2\right) - 2\epsilon L \left(\sin\theta\, d\phi_1 dt + \cos\theta\, d\phi_2 dt\right)$$

"FRW"   Static, Compact space   Vorticity

- Plug in a bounce $a(t)$ and compute

$$G_{\mu\nu} \xRightarrow{\text{infer}} T_{\mu\nu}$$

- Compute to $O(\epsilon^2)$, choose $n^\mu = (1, \frac{1}{a}, 0, 0, 0, 0, 0)$

for $L^2 \frac{\ddot{a}}{a} \ll 1 \Rightarrow T_{tt} + \frac{T_{xx}}{a^2} \simeq M_7^5 \left[\frac{\epsilon^2}{2L^2} - 2\frac{\ddot{a}}{a}\right]$

Can be > 0 !

NECV required in compact space:

- for $n^\mu = (1, 0, 0, 0, 0, n_6, 0)$ & $n^\mu n^\nu g_{\mu\nu} = 0$

  $\Rightarrow \quad n^\mu n^\nu T_{\mu\nu} = \frac{\epsilon}{L^2}\sin\theta - 3\frac{\ddot{a}}{a}$

  always negative somewhere

- Use Casimir (or curvature)

- Can show $T_{\mu\nu} - T_{\mu\nu}^{\text{Casimir}} = T_{\mu\nu}^{D}$

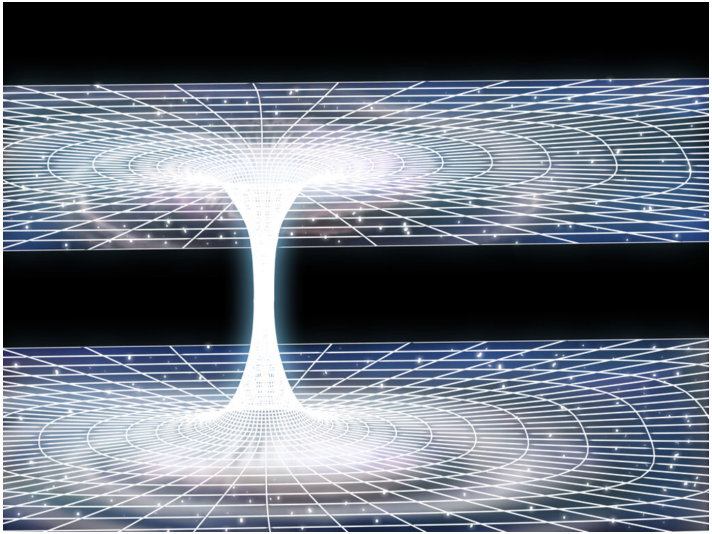
  where the difference, $T_{\mu\nu}^{D}$, can preserve NEC

What is this fluid (our current task)

Goal - A microscopic theory of $T^{D}_{\mu\nu}$

(we are close)

For $L^2 \frac{\ddot{a}}{a} << 1$, there must be a 4D effective theory with NECV fluid

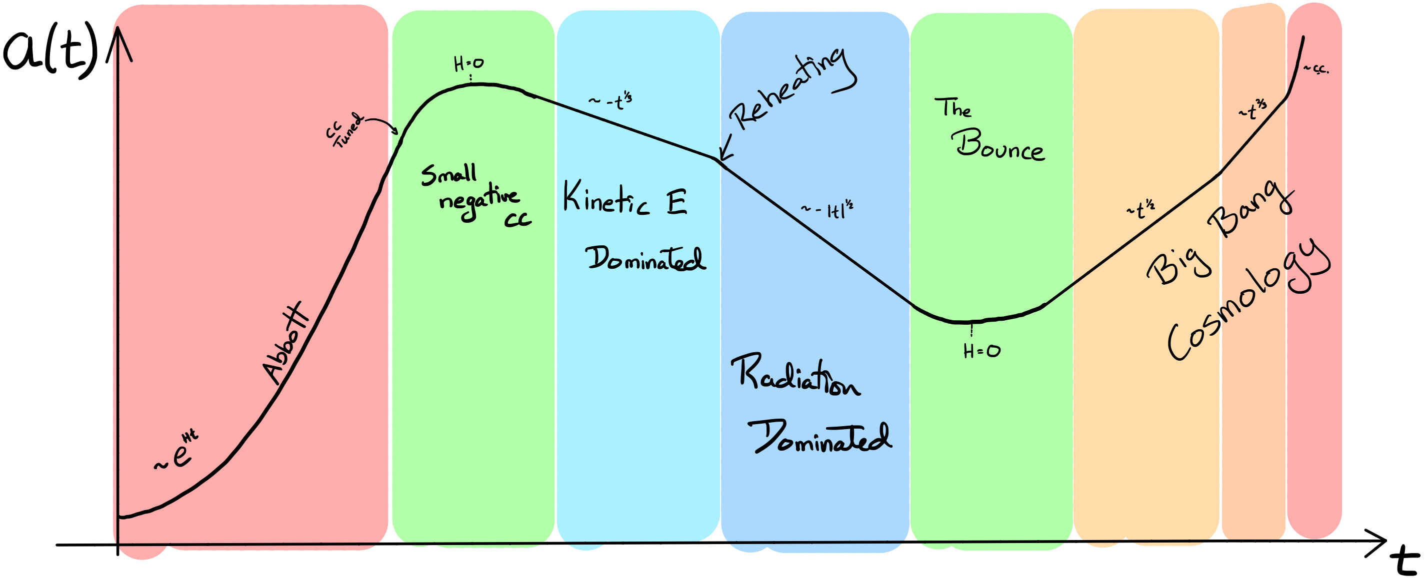


Once NECV is allowed, we can have real fun

Cosmological Consequences

- New Background Fluids
  - may have non-trivial perturbations
- New massless scalar – some non-gravitational coupling req'd.
- "Dark Energy" may be evolving
  (should explain why c.c. today is >0)
- Contraction phase may be involved w/ power spectrum ...

Summary



Thank You!